6. Use **Stokes's Theorem** to evaluate line integral $\int_C F.dr$, where $F(x,y,z) = x^2zi + xy^2j + z^2k$ and C is the curve of intersection of the plane $x+y+z=1$ and the cylinder $x^2+y^2=9$ oriented counterclockwise.

$x+y+z=1$

$\Rightarrow z = 1-x-y = g(x,y)$

Normal vector $N = -g_x i - g_y j + k$ 0.5

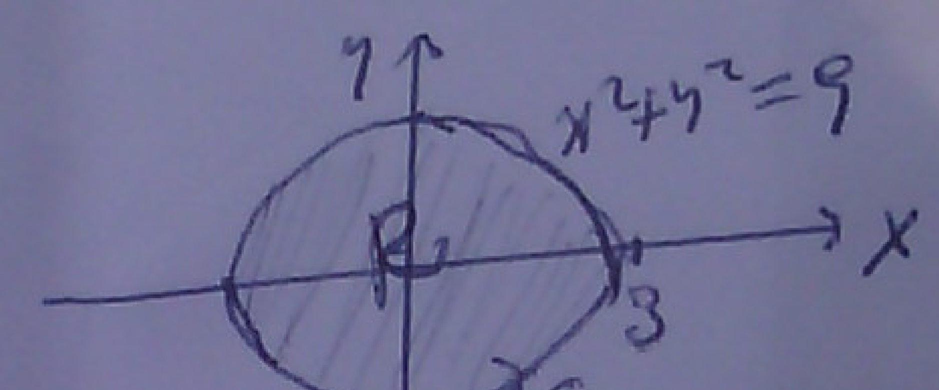
$= i + j + k$

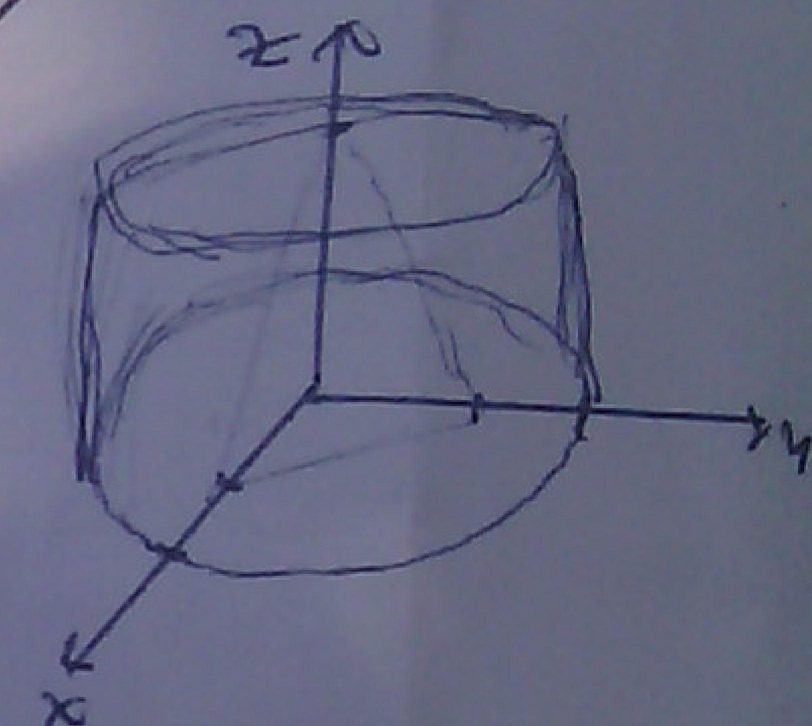
0.5

$\text{curl} F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2z & xy^2 & z^2 \end{vmatrix} = (0-0)i - (0-x^2)j + (y^2-0)k$

$= x^2j + y^2k$ 0.5

$\text{curl} F \cdot N = (x^2j + y^2k)\cdot(i+j+k) = x^2+y^2$





The region R is not simple, so in terms of polar coordinates:

$R = \{(r,\theta): 0 \le r \le 3,\ 0 \le \theta \le 2\pi\}$ 1

1

Line integral

$\int_C F.dr = \iint_R \text{curl}F \cdot N\,dA$ ... by stokes's Thm

$= \int_0^{2\pi}\int_0^3 [(r\cos\theta)^2 + (r\sin\theta)^2]\,r\,dr\,d\theta$ 0.5

$= \int_0^{2\pi}\int_0^3 r^3\,dr\,d\theta$

$= \int_0^{2\pi} \frac{r^4}{4}\Big|_0^3\,d\theta$

$= \int_0^{2\pi} \frac{81}{4}\,d\theta$

$= \frac{81}{4}\theta\Big|_0^{2\pi}$

$= \frac{162}{4}\pi = \underline{\frac{81}{2}\pi}$ 1